





UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

CLASS - 10

Question Paper Code : UN40109

KEY

1	2	3	4	5	6	7	8	9	10
D	С	В	С	D	D	А	В	В	С
11	12	13	14	15	16	17	18	19	20
С	В	D	В	А	В	С	С	С	С
21	22	23	24	25	26	27	28	29	30
D	D	С	В	В	D	А	D	А	С
31	32	33	34	35	36	37	38	39	40
B,C	A,C	A,C	B,C	A,B,C,D	В	С	С	D	А
41	42	43	44	45	46	47	48	49	50
С	В	С	D	В	С	А	D	С	D

SOLUTIONS

MATHEMATICS - 1

01. (D) Given in $\triangle ABC$

 $\angle B = 90^{\circ} \& AC = 3\sqrt{10} cm$

Let
$$AB = x \& BC = y$$

$$A = \frac{3\sqrt{10} \text{ cm}}{x}$$

$$B = \frac{3\sqrt{10} \text{ cm}}{y}$$

$$\therefore x^2 + y^2 = (3\sqrt{10})^2 = 90 \rightarrow (1)$$

Given $(3x)^2 + (2y)^2 = (9\sqrt{5})^2$ $\Rightarrow 9x^2 + 4y^2 = 405 \rightarrow (2)$ eq (2) - eq (1) × 4 $\Rightarrow (9x^2 + 4y^2) - (4x^2 + 4y^2) = 405 - 4 \times 90$ $5x^2 = 45$ $x^2 = \frac{45}{5} = 9$ $x = \sqrt{9} = 3$ $9 + y^2 = 90 \rightarrow (1)$ $y^2 = 90 - 9 = 81$

$$y = \sqrt{81} = 9$$

$$x + y = 3 + 9 = 12 \text{ cm}$$
02. (C) Construction: Join PA, PS & PC
AL² + BM² + CN² = AP² - PL² + BP² - PM²
+ CP² - PN²
= BP² - PL² + CP² - PM² + AP² - PN²
A
A
12 cm
B
B
A
A
C
= BL² + CM² + AN² = (3 cm)² + (4 cm)² +
(12 cm)²
= 9 cm² + 16 cm² + 144 cm² = 169 cm²
03. (B) 6 m 5 cm = 605 cm
20 m 35 cm = 2035 cm
HCF of 2035 cm and 605 cm
605)2035(3
1815
220)605(2
440
55)2100(20
1100
MCF = 55 cm
04. (C) (x² - 4x + 4) (x + 3) = (x - 2)² (x + 3)
(x² + 2x - 3)(x - 2) = (x + 3)(x - 1)(x - 2)
Common factors are (x - 2) (x + 3).
H.C.F. of given polynomals are (x - 2)
(x + 3).
05. (D)
05. (D)
05. (D)
05. (D)
07. (A)

tower and suppose it has been raised by h m. Then, С h m В A 40 m $\tan 45^\circ = \frac{AB}{40}$ or AB = 40 m $\tan 60^\circ = \frac{h + AB}{40}$ \Rightarrow h = 40 $\sqrt{3}$ – AB = 40 $(\sqrt{3}$ – 1)m Let P(2, 5) divides the Join of A(8, 2) and B(-6, 9) in the ratio $m_1 : m_2$ $\therefore P(2,5) = \left(\frac{m_1(-6) + m_2 \times 8}{m_1 + m_2}, \frac{9m_1 + 2m_2}{m_1 + m_2}\right)$ $\therefore \quad \frac{-6m_1 + 8m_2}{m_1 + m_2} = 2 \Longrightarrow -6m_1 + 8m_2 = 2m_1 + 2m_2$ $-6m_1 - 2m_1 = 2m_2 - 8m_2$ \neq 8m₁ = \neq 6m₂ $\frac{m_1}{m_2} = \frac{6}{8} = \frac{3}{4}$ $\therefore m_1 : m_2 = 3 : 4$ Given PQ = PR $\sqrt{(h+2)^2+16} = \sqrt{(h-4)^2+16}$ \Rightarrow h² + 4h + 20 = h² - 8h + 32

 \Rightarrow 12h = 12

 \Rightarrow h = 1

Let AB be the height of the incomplete

08. (B) Given
$$\triangle ADE \sim \triangle ABC \Rightarrow \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$

$$\Rightarrow \frac{1.2 \text{ cm}}{BC} = \frac{3 \text{ cm}^2}{7.5 \text{ cm} \text{ s}}$$

$$\Rightarrow BC = \frac{1.2^{0.6} \text{ cm} \times 5}{2 \text{ 1}}$$

$$= 3 \text{ cm}$$
09. (B) The two-digit number is of the form 7n + 3
First two-digit number will be for n = 1
i.e., 7 × 1 + 3 = 10
Last two-digit number will be for n = 1
i.e., 7 × 13 + 3 = 94
No. of terms = 13
Sum of all 13 terms

$$= \frac{13}{2} (10 + 94)$$

$$= 13 \times 52 = 676$$
10. (C) $\sec \theta + \tan \theta = \theta$
 $\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = p \text{ 1} + \sin \theta = p \cos \theta$
 $\sin \theta + \tan \theta = p,$
 $\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = p \text{ or } 1 + \sin \theta = p \cos \theta$
 $\Rightarrow (\sin \theta + \tan \theta)^2 = p^2,$
 $\Rightarrow \sec^2 + \tan^2 \theta + 2\sec \theta \tan \theta$
 $\Rightarrow p^2 + 1 = 2 \sec^2 \theta + 2\sec \theta \tan \theta$
 $\Rightarrow p^2 + 1 = 2 \sec^2 \theta (\sec \theta + \tan \theta)$
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 $\Rightarrow p^2 + 1 = 2 \sec^2 \theta (\sec^2 \theta + \tan^2 \theta)$
 $\Rightarrow \cos^2 = \frac{2p}{p^2 + 1}$

11. (C) Volume of the vessel

$$= \frac{1}{2} \times \frac{4}{3} \pi (R^{3} - r^{3})$$

$$= \frac{2}{3} \times \frac{22}{7} \left[\left(\frac{21}{2} \right)^{3} - 7^{3} \right] cm^{3}$$

$$= 44 \left(\frac{21^{2}}{2} - \frac{49}{3} \right)$$

$$= 44 \times \frac{931}{24} cm^{3} = \frac{10241}{6} cm^{3}$$

$$\therefore \text{ Weight of the vessel}$$

$$= \frac{10241 \times 10}{6} \text{ kg} = 17.07 \text{ k}$$
12. (B) Let co-ordinates of Q be (x, y) .

$$\frac{x - 2}{2} = 4; \frac{y + 9}{2} = 3$$

$$x = 10; y = -3$$

$$\therefore (10, -3) \text{ are the co-ordinates of Q.$$
13. (D) $X = \left(\frac{m_{1}x_{2} + m_{2}x_{1}}{m_{1} + m_{2}}, \frac{m_{1}y_{2} + m_{2}y_{1}}{m_{1} + m_{2}} \right)$

$$= \left(\frac{(2 \times -3) + 3 \times 7}{5}, \frac{2 \times 6 + 4 \times 3}{5} \right)$$

$$= \left(\frac{15^{3}}{5}, \frac{24}{5} \right)$$
14. (B) LHS = 3(sin\theta - cos\theta)^{4} + 6(sin\theta + cos\theta)^{2} + 4(sin^{6}\theta + cos^{6}\theta)
$$= 3(sin^{2}\theta + cos^{2}\theta - 2sin\theta cos\theta) + 4[(sin^{2}\theta + cos^{2}\theta)^{3} - 3sin^{2}\theta cos^{2}\theta (sin^{2}\theta + cos^{2}\theta)]$$

$$= 3(1 - 2 \sin\theta cos\theta)^{2} + 6(1 + 2 \sin\theta cos\theta)$$

$$\sqrt{3}x^{2} + 12x - 2x - 8\sqrt{3} = 0$$

$$\sqrt{3}x(x + 4\sqrt{3}) - 2(x + 4\sqrt{3}) = 0$$

$$(x + 4\sqrt{3})(\sqrt{3}x - 2) = 0$$

$$x + 4\sqrt{3} = 0 \quad (\text{or}) \quad \sqrt{3}x - 2 = 0$$

$$x = -4\sqrt{3} \quad (\text{or}) \quad x = \frac{2}{\sqrt{3}}$$
(C)
$$\cos\theta = \frac{10}{20} = \frac{1}{2} \Rightarrow \theta = 60^{\circ}$$

$$\angle \text{PLK} = 180^{\circ} - 90^{\circ} - 60^{\circ} = 30^{\circ}$$

$$\angle \text{NLM} = 180^{\circ} - 90^{\circ} - 30^{\circ} = 60^{\circ}$$

$$\cos \angle \text{NLM} = \cos 60^{\circ} = \frac{\text{NL}}{\text{LM}} = \frac{\text{NL}}{6}$$

$$\Rightarrow \frac{1}{2} = \frac{\text{NL}}{6} \Rightarrow \text{NL} = 3 \text{ cm}$$

$$\therefore \quad \text{MN} = \sqrt{\text{M L}^{2}} = \sqrt{6^{2} - 3^{2}} = \sqrt{27}$$
Hence, MN = $3\sqrt{3}$ cm
(C) Join O to P and Q. Join P to R. Draw SP
$$\perp \text{OQ}.$$
Now SP = QR, as they are opposite sides
of rectangle PRQS.
OP = 8 cm + 4 cm = 12 cm
OS = 8 cm - 4 cm = 4 cm

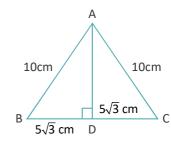
$$\therefore \quad \text{SP} = \sqrt{\text{OP}^{2} - \text{OS}^{2}}$$

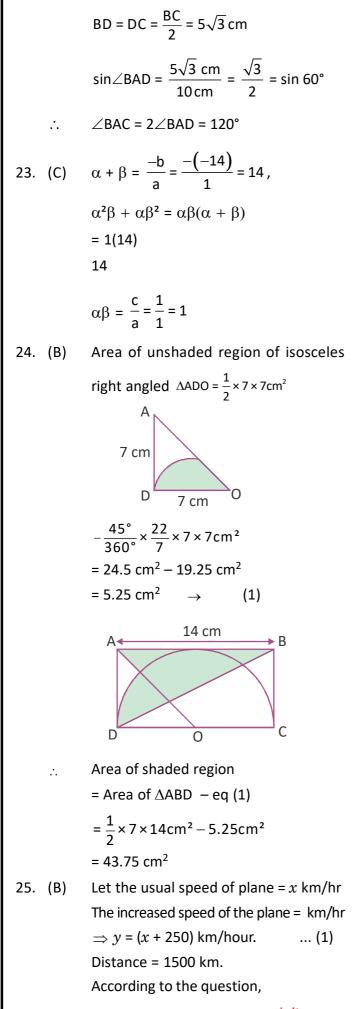
$$= \sqrt{12^{2} - 4^{2}} \text{cm} = 8\sqrt{2} \text{ cm}$$

$$\therefore \qquad QR = 8\sqrt{2} \text{ cm}$$
(D) The required distance =

$$\sqrt{\left[\left(\sqrt{3} + 1\right) - \left(\sqrt{3} - 1\right]\right]^{2} + \left[\left(\sqrt{2} - 1\right) - \left(\sqrt{2} + 1\right)\right]^{2}}$$

$$= \sqrt{(2)^{2} + (2)^{2}} = 2\sqrt{2}$$
(D) Construct AD \perp BC





(Scheduled time) - (time in increasing the speed) = 30 minutes $\frac{1500}{x} - \frac{1500}{y} = \frac{1}{2}$... (2) $\mathsf{Time} = \frac{\mathsf{Distance}}{\mathsf{Speed}}$ $\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2} \quad [\therefore \text{ From (1)}]$ $\frac{1500x + 375000 - 1500x}{x(x + 250)} = \frac{1}{2}$ \Rightarrow x(x + 250) = 750000 \Rightarrow $x^2 + 250x - 750000 = 0$ \Rightarrow $x^2 + 1000x - 750x - 750000 = 0$ \Rightarrow x (x + 1000) - 750 (x + 1000) = 0 \Rightarrow (x - 750) (x + 1000) = 0 \Rightarrow *x* = 750 or *x* = - 1000 \Rightarrow But speed can never be negative Hence, the usual speed = 750 km/hr 26. (D) The first 'n' even numbers are 2, 4, 6, ..., 2n $\therefore S_n = \frac{n}{2}(2+2n) = n(n+1)$ OC = OD and OA = OP = OB 27. (A) OP = 1 mPC = 1m $OC = \sqrt{2} m$ AC = OC - OA÷ $=(\sqrt{2}-1)$ m and AC + AP = $(\sqrt{2} - 1) + 1$ $=\sqrt{2}$ = 1.414 m

28. (D) a and b are the roots of
$$x^2 + px + 1 = 0$$

 $\Rightarrow \alpha + \beta = -p, \alpha\beta = 1$
 $\gamma \text{ and } \delta \text{ are the roots of } x^2 + qx + 1 = 0$
 $\Rightarrow \gamma\delta = 1$
 $\gamma^2 + q\gamma + 1 = 0 \Rightarrow \gamma^2 + 1 = -q\gamma$
 $\delta^2 + q\delta + 1 = 0 \Rightarrow \delta^2 + 1 = -q\delta$
 $(\alpha - \gamma) (\beta - \gamma) (\alpha + \delta) (\beta + \delta)$
 $= [\alpha\beta - \gamma(\alpha + \beta) + \gamma^2]$
 $[\alpha\beta + \delta(\alpha + \beta) + \delta^2]$
 $= (1 + p\gamma + \gamma^2)(1 - p\delta + \delta^2)$
 $= (-q\gamma + p\gamma)(-p\delta - q\delta)$
 $= -\gamma(q - p) \times \delta(p + q)$
 $= \gamma\delta(q^2 - p^2)$
29. (A) $\sin 0^\circ + \cos 30^\circ - \tan 45^\circ$
 $+ \csc 60^\circ + \cot 90^\circ$
 $= 0 + \frac{\sqrt{3}}{2} - 1 + \frac{2}{\sqrt{3}} + 0$
 $= \frac{3 - 2\sqrt{3} + 4}{2\sqrt{3}}$
 $= \frac{7\sqrt{3} - 6}{6}$
30. (C) $ABC, \angle BAC = 90^\circ$
 $[\because A \text{ tangent is perpendicular to radius]}$
 $AC^2 = \sqrt{BC^2 - AB^2}$
 $= \sqrt{15.25^2 - (2.75)^2}$
 $= \sqrt{232.5625 - 7.5625}$
 $= \sqrt{225} \text{ cm}$
 $= 15 \text{ cm}$

MATHEMATICS - 2 31. (B, C) In $\triangle ABC$, DE||BC $\Rightarrow \frac{AD}{BC} = \frac{AE}{CE} \Rightarrow \frac{3x-2}{7x-5} = \frac{5x-4}{5x-3}$ $\Rightarrow x = 1 \text{ or } \frac{7}{10}$ 32. (A, C) x + 1) $x^{3} - 3x - 2(x^{2} - x - 2)$ $\frac{x^{3} + x^{2}}{-x^{2} - 3x - 2}$ $\frac{-x^2 - 0x - 2}{-x^2 - x}$ $\frac{-2x-2}{0}$ $x^{3}-3x-2 = (x + 1) (x^{2}-x-2)$ = (x + 1) (x + 1) (x - 2)33. (A, C) Given Sin(A + B) = $\frac{\sqrt{3}}{2}$ = Sin60° $\therefore A + B = 60^{\circ} \longrightarrow (1)$ Given $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30^{\circ}$ $\therefore A - B = 30^{\circ} \rightarrow (2)$ eq (1) + (2) \Rightarrow A + B' + A - B' = 60° + 30° = 90° $2A = 90^{\circ} \Rightarrow \angle A = 45^{\circ}$ $45^{\circ} + B = 60^{\circ}$ $B = 60^{\circ} - 45^{\circ}$ B = 15° 34. (B, C) The given relation can be written as $(m + 2) \tan \theta + (2m - 1) = (2m + 1) \sec \theta$ \Rightarrow (m + 2)² tan² θ + 2(m + 2) (2m - 1)tan θ + $(2m - 1)^2$ $= (2m + 1)^2 (1 + \tan^2 \theta)$ \Rightarrow [(m + 2)² - (2m + 1)²] tan² θ + 2(m + 2) $(2m-1) \tan \theta + (2m-1)^2 - (2m+1)^2 = 0$

 \Rightarrow 3(1-m²) tan² θ + (4m² + 6m - 4) tan θ - 8m = 0

 \Rightarrow (3 tan θ - 4) [(1 - m²) tan θ + 2m] = 0 which is true if $\tan \theta = \frac{4}{3}$ or $\tan \theta$ $= 2m/(m^2 - 1)$ 35. (A, B, C, D) ABCD is a cyclic quadrilateral $\therefore \angle A + \angle C = 180^{\circ} \& \angle B + \angle D = 180^{\circ}$ $2x - 3^{\circ} + 2y + 17^{\circ} = 180^{\circ}$ $y + 7^{\circ} + 4x - 9^{\circ} = 180^{\circ}$ $2x + 2y + 14^{\circ} = 180^{\circ}$ $4x + y = 182^{\circ} \rightarrow 2$ $2x + 2y = 180^{\circ} - 14^{\circ}$ $2(x + y) = 166^{\circ}$ $x + y = \frac{166^{\circ}}{2} = 83^{\circ} \rightarrow (1)$ Solving eq (1) & (2) we get $x = 33^{\circ} \& y = 50^{\circ}$ $\therefore \angle A + \angle B = 2x - 3^{\circ} + y + 7^{\circ} = 63^{\circ} + 50^{\circ} + 7^{\circ} = 120^{\circ}$ $\angle A = 2x - 3 = 2 \times 33^{\circ} - 3^{\circ} = 66^{\circ} - 3^{\circ} = 63^{\circ}$ $\angle B = y + 7^{\circ} = 50^{\circ} + 7^{\circ} = 57^{\circ}$ $\angle C = 2y + 17^{\circ} = 100^{\circ} + 17^{\circ} = 117^{\circ}$ $\angle D = 180^{\circ} - \angle B = 180^{\circ} - 57^{\circ} = 123^{\circ}$ $\angle A + \angle D = 63^{\circ} + 123^{\circ} = 186^{\circ}$ $\angle A + \angle B = 63^{\circ} + 57^{\circ} = 120^{\circ}$ $\angle B + \angle C = 57^{\circ} + 117^{\circ} = 174^{\circ}$ REASONING 36. (B) $P \xrightarrow{+2} R \xrightarrow{+2} T \xrightarrow{+2} V \xrightarrow{+2} X$ $3 \xrightarrow{+2} 5 \xrightarrow{+3} 8 \xrightarrow{+4} 12 \xrightarrow{+5} 17$ $C \xrightarrow{+3} F \xrightarrow{+3} I \xrightarrow{+3} L \xrightarrow{+3} O$ 37. (C) The answer figure is as shown below: Hence answer is (C) Column one : $4 \rightarrow 4(2)^2 = 416$ 38. (C) Column two : 7 \rightarrow 7(7)² = 749 Column three : $3 \rightarrow 3(3)^2 = 309$ Column four : 2 \rightarrow 2(2)² = 204 Hence correct answer is option (C)



40. (A) The number of circles increase by one each time.

The shading of the new circle alternates from shading to unshaded.

When the circle is added, the remaining circles move around the edge of the box in an anti clock wise direction.

The missing box must look like the below



41. (C) There are 14 squares in the given figure.

1. AFMG	2. GHMN	3. BHNI
4. EFMI	5. KLMN	6. IJKN
7. CQKJ	8. LKQP	9. DELP
10. AHKE	11. GBJL	12. CPMI
13. DFNQ	14. ABCD.	

42. (B) Since the total number of dots on opposite face is always 7, therefore, 1 dot appears opposite 6 dots, 2 dots appear opposite 5 dots and 3 dots appear opposite 4 dots.

Fig. A, C, D are wrong since,

1 dot cannot appear adjacent to 6 dots.

3 dots cannot appear adjacent to 4 dots.

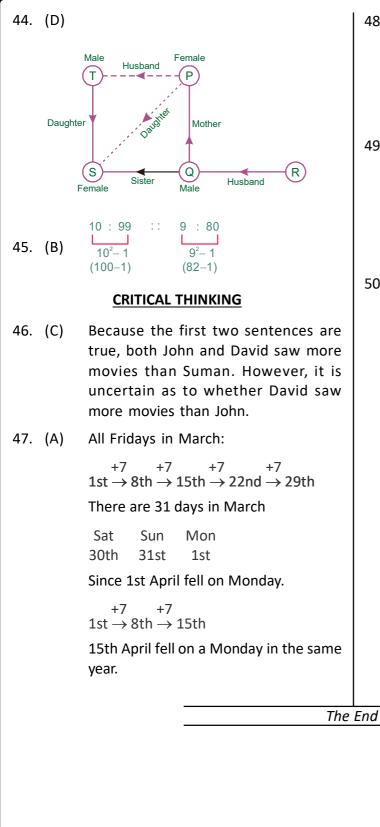
and 2 dots cannot appear adjacent to 5 dots.

43. (C) The new alphabet series after deleting every alternate letter starting from B is

A C E G I K M O Q S U W Y

Now 3 + 4 = 7 (left to right)

The seventh letter from the left is M.



- 48. (D) Closing the schools for a week and the parents withdrawing their wards from the local schools are independent issues, which must have been triggered by different individual causes.
- 49. (C) Bench I P T S Bench II U Q Bench III V R \Box = Boy \bigcirc = Girl

QRS are group of girls.

50. (D) The first sentence makes this statement true. There is no support for choice a. The passage tells us that the spa vacation is more expensive than the island beach resort vacation, but that doesn't necessarily mean that the spa is overpriced; therefore, choice b cannot be supported. And even though the paragraph says that the couple was relieved to find a room on short notice, there is no information to support choice c, which says that it is usually necessary to book at the spa at least six months in advance.